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**Mathematical Model of A Beam in Space** Approved

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**Mathematical Model of A Beam in Space**

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BY  
DONALD B. HANKS, JR.

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A THESIS

Submitted to the Faculty of the Graduate School of the Creighton University in  
Partial Fulfillment of the Requirements for the Degree of Master of Science in  
the Department of Mathematics

---

Omaha, 1997



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## INTRODUCTION

Today, as our world stands on the edge of the twenty-first century, we prepare to take a giant leap towards the colonization of space with the construction of space station Freedom. Assembling a large space structure, like Freedom, in orbit is a technical and logistical triumph, but it is a short lived victory. Getting the station in orbit is half the battle, and keeping it there is the other. All large space structures in earth orbit are acted upon by the remnants of our atmosphere and the earth's gravity (although small at higher altitudes). This results in drag slowing the structures down and causing them to lose altitude. If no corrective action is taken to boost the structure to a higher orbit, it will reenter the earth's atmosphere. This exact scenario has already occurred once with the space station Skylab [1].

It is obvious that for a large space structure to remain in orbit that some method must be used to periodically boost it back to higher orbits. There are different methods that can be employed. For instance an external ship, such as the shuttle, could use its own maneuvering rockets to boost the station. Another alternative is to have maneuvering rockets on the station itself which can be fired to move to higher altitudes. It is expected that a large space structure acting under the forces of maneuvering rockets (regardless of method used) will experience a rigid translation in a direction opposite the force applied, vibrations around its center of mass, as well as a physical deformation of its structure. The goal of this thesis is to develop a mathematical model, using differential

equations, of a large space structure acting under the force of these maneuvering rockets. This model could then be used to analyze the structural responses of a large space structure generated by the firing of the maneuvering rockets. For the purposes of this thesis only the second case, where the rockets are part of the large space structure, will be considered.

The process that will be used to accomplish this is to take a model that is used on earth and make necessary changes in assumptions, boundary conditions, etc., to adapt it to space. Chapter one presents the mathematical development of the model from an Euler beam model used on earth. Chapter two shows the numerical analysis of the solutions of the mathematical model, and discusses the responses of the model to the forces encountered. Chapter three examines the vibrations, caused by the force of the maneuvering rockets, to ensure that they are bounded. Finally chapter four presents conclusions based upon the results of the previous chapters.



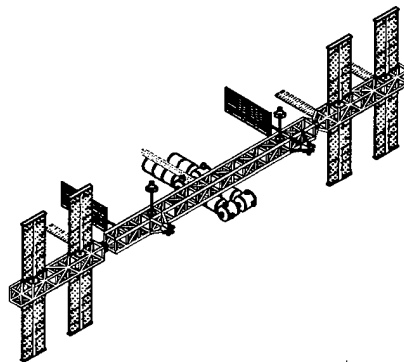
## CHAPTER 1

### MATHEMATICAL DEVELOPMENT OF THE MODEL

This chapter deals with the development of a mathematical model to represent a large space structure. First, we will select a physical structure that will represent a large space structure, then we will set about describing it mathematically.

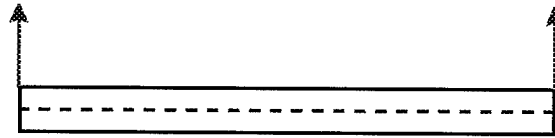
Before any mathematical equations can be developed we have to know what it is we are attempting to model. Pictured below is a typical space station type structure that we want to model.

Fig 1-1



Thus, our first step is to choose a physical model to represent a large space structure like the one pictured above. Our main concerns about this choice are 1) does our physical model adequately represent a large space structure, and 2) can a mathematical model be developed to describe this physical model. Taking these concerns into account the first structure that comes to mind is that of a simple beam (representing the space structure) floating in space, with forces applied at the ends (representing the force of the maneuvering rockets) as seen in the picture below.

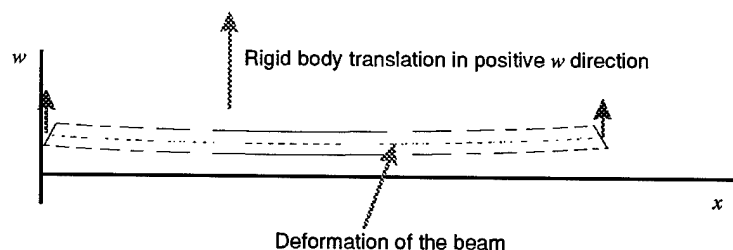
Fig 1-2



On earth simple beams are used extensively in the theory of elasticity and structural dynamics and react to forces similar to what we are trying to model . Given the extensive work done with beams on earth we have a high degree of confidence that we can accurately mathematically model this structure in space.

Now that a physical model has been chosen we can begin the process of mathematically describing it. The first step in this process is identifying exactly what we are trying to do mathematically. As mentioned earlier we want to model a maneuvering rocket type force acting on a large space structure (represented by a simple beam with a point force applied at each end), and the response of the beam to this force. It is reasonable to expect the beam to experience, 1) a rigid translation of the beam in the positive  $w$  direction, 2) vibrations around the center of mass of the beam, 3) as well as a deformation of the beam in response to the force caused by a maneuvering rocket burn (see Fig 1-3).

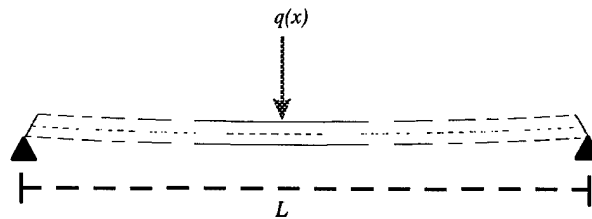
Fig 1-3



These three responses will be the main areas that our mathematical model must address.

Now that we have chosen a physical structure, and identified the goals of our mathematical model we can start doing some math. We begin by looking at existing models of beams acting under forces. Immediately we turn to the theory of elasticity and the model of a simply supported beam of length,  $L$ , acting under a vertical load  $q(x)$ , as shown in Fig 1-4.

Fig 1-4



The dotted line running through the middle of the beam is known as the *center line*. This center line becomes deformed under the action of the vertical load  $q(x)$ , and is then referred to as the *elastic curve* of the beam [2]. This center line/elastic curve will become important as we begin discussing the assumptions made about this beam. The differential equation representing the equilibrium of this beam is derived from the Bernoulli-Euler law which states that the curvature  $K$  of the elastic curve is proportional to the bending moment  $M$  [2] and is given by,

$$(1.1) \quad K = \frac{M}{EI}$$

where  $E$  is the Young's modulus of elasticity and  $I$  is the area moment of inertia. Using equation (1.1) the fourth-order partial differential equilibrium equation is derived to be,

$$(1.2) \quad EIw^{[4]} = q(x)$$

where  $w(x)$  is the equation of the elastic curve of the beam as a function of the distance,  $x$ , along the center of the beam from the left endpoint (represents the deflection of the beam), and  $q(x)$  is the force acting on the beam [3]. This model is based on several assumptions the greatest of which comes from Von Karman. This assumption states that any vector drawn to a point off the centerline, that is normal to the centerline before deformation will remain normal to the centerline after deformation has occurred, and the distance to this point will be preserved [4]. The other assumption is that the ends of this beam are fixed and hinged, which leads to the following boundary conditions [3],

$$(1.3.a) \quad w(0) = w(L) = 0$$

$$(1.3.b) \quad w''(0) = w''(L) = 0$$

We also expect vibrations around the equilibrium solution of equation 1.2 to occur after a force is applied. Therefore using D'Alembert's principle [4] we add in an acceleration term into the equilibrium equation giving us,

$$(1.4.a) \quad EIw_{xxxx} + \rho w_{tt} = q(x)$$

where  $w$  is now a function of distance,  $x$ , and time,  $t$ , and  $w_{tt}$  is the acceleration of the beam with respect to time, and,  $\rho$ , is the density of the beam. Dividing both sides by  $\rho$  gives us,

$$(1.4.b) \quad Gw_{xxxx} + w_{tt} = \tilde{q}(x)$$

where  $G = \frac{EI}{\rho}$ , and  $\tilde{q}(x) = \frac{q(x)}{\rho}$ .

Equation 1.4 is correct for a beam on earth, but since we want to examine a beam floating free in space, modifications must be made to this equation. Since the beam is floating free the ends are no longer fixed, as on earth, and our boundary conditions change. The third partial derivative of  $w$  with respect to  $x$ , which is the shear term, will be 0 at the endpoints, and we cannot assume  $w(0,t) = w(L,t) = 0$ . Therefore, our new boundary conditions are,

$$(1.5.a) \quad w_{xx}(0,t) = w_{xx}(L,t) = 0$$

$$(1.5.b) \quad w_{xxx}(0,t) = w_{xxx}(L,t) = 0$$

The status of the endpoints not only affects our boundary conditions, but our solution as well. On earth with fixed endpoints we would expect a solution to 1.4 to consist of vibrations around the equilibrium solution of the beam and a deformation of the beam. Now with endpoints not fixed the solution will consist of a rigid body translation of the center of mass of the beam, vibrations around the center of mass, and a deformation of the beam. In chapter 2 we will be looking for a solution to 1.4 of the form,

$$(1.6) \quad w(x,t) = u(x,t) + v(t)$$

where  $v(t)$  represents the rigid body translation of the beam. According to Newton's law of motion,  $F=ma$ , the acceleration,  $v''(t)$ , is given by

$$(1.7.a) \quad mv''(t) = \int_0^L q(x,t)dx$$

where  $m = \rho L$  (for our case), and  $\int_0^L q(x,t)dx$  is the force acting on the beam

integrated over the length of the beam, and  $v(t)$  is of the form  $at^2 + bt + c$  (where  $a, b, c$  are constants). Dividing both sides of 1.7.a by  $m = \rho L$  we obtain

$$(1.7.b) \quad v''(t) = \frac{1}{L} \int_0^L \tilde{q}(x, t) dx$$

where  $\tilde{q}(x, t) = \frac{q(x, t)}{\rho}$ . If we assume that  $v(0) = 0$ , and  $v'(0) = 0$ , then  $v(t) = at^2$ .

Substituting 1.6 into 1.4 we get the following equation which mathematically models a beam in space,

$$(1.8) \quad u_{tt} + Gu_{xxxx} = \tilde{q}(x) - \frac{1}{L} \int_0^L \tilde{q}(x, t) dx = f(x)$$

where on the right side we are subtracting out our rigid body translation so that we are left with just deformation and vibrations. In effect we have changed to a new coordinate system now centered on the center of mass of the beam. We expect the general solution of 1.8 to consist of an equilibrium solution as well as some function representing the vibrations around this equilibrium solution. In general this solution,  $u(x, t)$ , is given by,

$$(1.9) \quad u(x, t) = u_0(x) + z(x, t)$$

where  $u_0(x)$  is the equilibrium solution, and  $z(x, t)$  is a function describing the vibrations around  $u_0(x)$ . Substituting 1.9 into 1.8 we get,

$$(1.10) \quad z_{tt} + Gz_{xxxx} = 0$$

and,

$$(1.11) \quad Gu_0^{[4]}(x) = f(x)$$

Ultimately we want to examine  $z(x,t)$  to ensure that the vibrations around this equilibrium solution are bounded.

Now that we have developed a mathematical model, we must approximate the solutions. In the next chapter numerical calculations and analysis used to approximate the general solution,  $u(x,t)$ , to our model are presented.

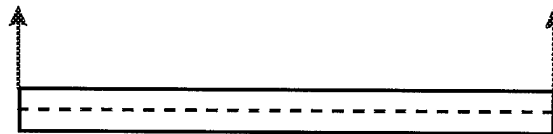
## CHAPTER 2

### NUMERICAL ANALYSIS OF SOLUTIONS

As already stated the general solution to our model, given by equation 1.8, consists of two parts. The equilibrium solution,  $u_0(x)$ , and the vibrations around this equilibrium solution,  $z(x,t)$ . In this chapter we will first examine the equilibrium solution,  $u_0(x)$ , then  $z(x,t)$ .

We want to find an equilibrium solution,  $u_0(x)$ , to equation 1.9. which represents the deflection of the beam caused by the force applied at the ends. It is useful at this point to recall that in equation 1.8 the rigid body motion was subtracted out and will not be considered in these calculations. Recall, from chapter 1 that we are representing our space station type structure as a beam with a point force,  $f_0(x)$ , applied upwards at each end of the beam (representing the force of maneuvering rockets) (see Fig 2-1).

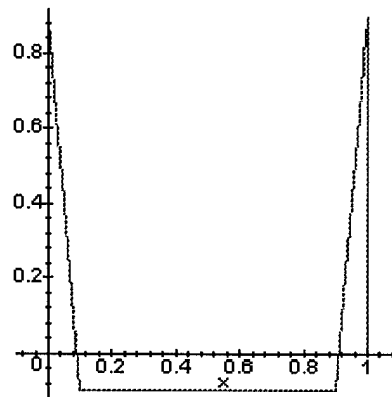
Fig 2-1 Beam in Space Model



For the purposes of this thesis we specify the length of our beam to be  $L=1$ . We choose as our  $f_0(x)$  a linear force acting over an interval of length  $\varepsilon > 0$  near each endpoint (Fig 2-2).



Fig 2-2 Force Acting Over An Interval



We then constructed the piecewise function below to represent  $f_0(x)$ , such that it

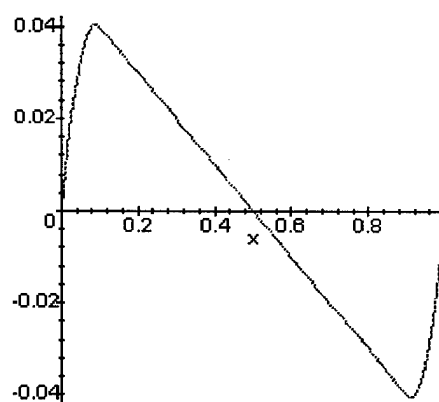
$$(2.1) \quad f_0(x) = \begin{cases} 0.9 - 10x & 0 \leq x \leq 0.1 \\ -0.1 & 0.1 \leq x \leq 0.9 \\ -9.1 + 10x & 0.9 \leq x \leq 1.0 \end{cases}$$

was continuous and its fourth derivative existed everywhere. Substituting in 2.1 into 1.11 we get,

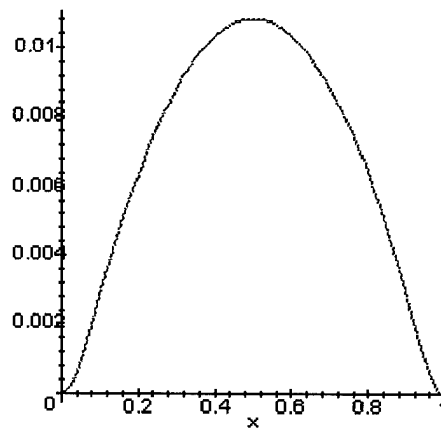
$$(2.3) \quad Gu_0^{[4]}(x) = f_0(x)$$

the equation that we will solve for  $u_0(x)$ . For the purposes of our analysis we will assume the constant,  $G = \frac{EI}{\rho} = 1$ . Using the Maple V, computer algebra system, with the stated boundary conditions, equation 2.3 was integrated four times (with respect to  $x$ ) to find  $u_0(x)$ . The numerical and graphical results from these integrations are listed below:

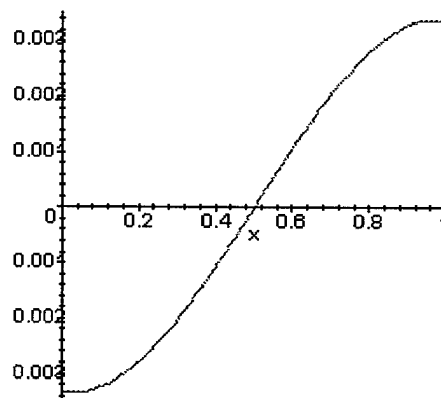
$$(2.4) \quad u_{xxx} = \begin{cases} 0 & x = 0 \\ 0.9x - 5x^2 & 0 \leq x \leq 0.1 \\ -0.1x + 0.05 & 0.1 \leq x \leq 0.9 \\ -9.1x + 5x^2 + 4.1 & 0.9 \leq x \leq 1.0 \\ 0 & x = 1 \end{cases}$$

Fig 2-3  $u_{xxx}$ 

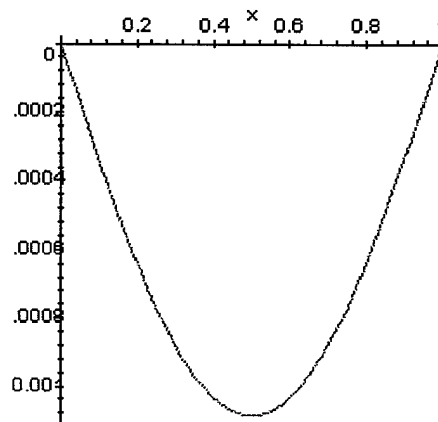
$$(2.5) \quad u_{xx} = \begin{cases} 0 & x = 0 \\ \frac{0.9}{2}x^2 - \frac{5}{3}x^3 & 0 \leq x \leq 0.1 \\ \frac{-0.1}{2}x^2 + 0.05x - 0.00167 & 0.1 \leq x \leq 0.9 \\ \frac{-9.1}{2}x^2 + \frac{5}{3}x^3 + 4.1x - 1.2167 & 0.9 \leq x \leq 1.0 \\ 0 & x = 1 \end{cases}$$

Fig 2-4  $u_x$ 

$$(2.6) \quad u_x = \begin{cases} 0 & x = 0 \\ \frac{0.9}{6}x^3 - \frac{5}{12}x^4 - 0.00335890 & 0 \leq x \leq 0.1 \\ -\frac{0.1}{6}x^3 + \frac{0.05}{2}x^2 - 0.00167x - 0.0033317 & 0.1 \leq x \leq 0.9 \\ -\frac{9.1}{6}x^3 + \frac{5}{12}x^4 + \frac{4.1}{2}x^2 - 1.2167x + 0.270085 & 0.9 \leq x \leq 1.0 \\ 0 & x = 1 \end{cases}$$

Fig 2-5  $u_x$ 

$$(2.7) \quad u_0 = \begin{cases} 0 & x = 0 \\ \frac{0.9}{24}x^4 - \frac{5}{60}x^5 - 0.00335890x & 0 \leq x \leq 0.1 \\ -\frac{0.1}{24}x^4 + \frac{0.05}{6}x^3 - \frac{0.00167}{2}x^2 - 0.0033317x - 0.00000084 & 0.1 \leq x \leq 0.9 \\ -\frac{9.1}{24}x^4 + \frac{5}{60}x^5 + \frac{4.1}{6}x^3 - \frac{1.2167}{2}x^2 + 0.270085x - 0.049235 & 0.9 \leq x \leq 1.0 \\ 0 & x = 1 \end{cases}$$

Fig 2-6  $u_0$ 

Equation 2.7 satisfies our boundary conditions and is an adequate equilibrium solution,  $u_0(x)$ . Now that we have an equilibrium solution we turn our attention toward solving for the vibration function,  $z(x,t)$ .

The function  $z(x,t)$  describing the vibrations of the beam will have a solution of the form,

$$(2.8) \quad z(x,t) = \sum_{j=1}^{\infty} X_j(x)T_j(t)$$

It is known that  $X_j(x)$  and  $T_j(t)$  have the following forms,

$$(2.9) \quad X_j(x) = A_j \sinh(\omega_j x) + B_j \cosh(\omega_j x) + C_j \sin(\omega_j x) + D_j \cos(\omega_j x)$$

$$(2.10) \quad T_j(t) = E_j \sin(\omega_j t) + F_j \cos(\omega_j t)$$

where  $A_j, B_j, C_j, D_j, E_j$ , and  $F_j$  are constant coefficients and the  $\omega_j$ 's are the eigenvalues of the functions. There will be an infinite number of eigenvalues that will satisfy these functions for our boundary conditions. We will only find the five of eigenvalues for the purpose of this thesis. Using our boundary conditions from Chapter 1,

$$(1.5.a) \quad w_{xx}(0, t) = w_{xx}(L, t) = 0$$

$$(1.5.b) \quad w_{xxx}(0, t) = w_{xxx}(L, t) = 0$$

and equation 2.9 we derive the following four equations,

$$(2.11.a) \quad X''(0) = B_j \omega_j^2 - D_j \omega_j^2$$

$$(2.11.b) \quad X''(L) = A_j \omega_j^2 \sinh(\omega_j L) + B_j \omega_j^2 \cosh(\omega_j L) - C_j \omega_j^2 \sin(\omega_j L) - D_j \omega_j^2 \cos(\omega_j L)$$

$$(2.11.c) \quad X'''(0) = A \omega^3 - C \omega^3$$

$$(2.11.d) \quad X'''(L) = A \omega^3 \cosh(\omega L) + B \omega^3 \sinh(\omega L) - C \omega^3 \cos(\omega L) + D \omega^3 \sin(\omega L)$$

which we will use to determine the various values of  $\omega_j$ . In order to find  $\omega_j$  we will put equations 2.11.a-d into a matrix,  $M$ , find the determinant of  $M$ , and then solve the equation  $\text{Det}(M)=0$ . The Maple V computer algebra system was used to carry out these operations, the results are below:

$$M = \begin{bmatrix} 0 & \omega^2 & 0 & -\omega^2 \\ \omega^2 \sinh(\omega L) & \omega^2 \cosh(\omega L) & -\omega^2 \sin(\omega L) & -\omega^2 \cos(\omega L) \\ \omega^3 & 0 & -\omega^3 & 0 \\ \omega^3 \cosh(\omega L) & \omega^3 \sinh(\omega L) & -\omega^3 \cos(\omega L) & -\omega^3 \sin(\omega L) \end{bmatrix}$$

$$\text{Det}(M) = -2\omega^{10} \sinh(\omega L) \sin(\omega L) - 2\omega^{10} \cos(\omega L)^2 + 2\omega^{10} \cosh(\omega L) \cos(\omega L)$$

Setting this determinant equal to zero the first five values of  $\omega_j$  were found and are listed below,

$$(2.12) \quad \omega = \begin{cases} 4.730040745 \\ 7.853204624 \\ 10.99560784 \\ 14.13716549 \\ 17.27875966 \end{cases}$$

Now we will demonstrate how these specific values of  $\omega_j$  can be used to find the coefficients  $A_j, B_j, C_j$ , and  $D_j$ . Using numerical approximation techniques values for the coefficients  $A_j, B_j, C_j$ , and  $D_j$  were found for the first five values of  $\omega_j$ . The table below lists these results:

TABLE 2-1

$\omega_j$	COEFFICIENT MATRIX [A, B, C, D]
4.730040745	[1, -1.017809412, 1.000000005, -1.017809507]
7.853204624	[1, -.9992232934, .9999999613, -.9992216947]
10.99560784	[1, -1.000033531, .9999951588, -.9994589301]
14.13716549	[1, -.9999985491, 1.000003425, -1.000704009]
17.27875966	[1, -1.000000066, 1.000208350, -1.061150339]

Now we know that velocity will equal 0 at  $t=0$ . Taking the first derivative of equation 2.10 we get,

$$(2.13) \quad T_j'(t) = E_j \omega \cos(\omega_j t) - F \omega_j \sin(\omega_j t)$$

Setting equation 2.13 equal to 0 and evaluating at  $t=0$  we get,

$$(2.14) \quad T_j'(t) = E_j = 0$$

and therefore  $E_j=0$ . We could also find  $F_j$  using numerical approximation techniques and initial conditions. However, for the purposes of this thesis we do not need to calculate it.

## CHAPTER 3

### BOUNDEDNESS OF VIBRATION

Developing a mathematical model to analyze how a large space structure reacts to certain forces is important. It allows us to find the specific responses caused by certain forces. Ultimately however we are more concerned with whether there is a limit to these responses. Specifically, we are concerned with the vibrations of a large space structure that are caused by maneuvering rocket type forces. If these vibrations have no limit, no maximum value, they will continue to increase without bound. Eventually a structural failure will occur resulting in a loss of equipment and human life. We have examples on earth of how catastrophic unbounded vibrations can be. The Tacoma Narrows bridge disintegrated as a result of unbounded vibrations caused by the wind [5]. It is obvious then that proving the vibrations of a large space structure to be bounded is of the utmost importance. In this chapter we will show that the vibrations are in fact bounded.

In order to prove boundedness we will use what is known as the *Energy Method* [6]. We start by multiplying equation 1.10 by  $z_t$ , and integrating with respect to  $x$ , to get,

$$(3.1) \quad \int_0^L z_{tt} z_t dx + \int_0^L z_{xxx} z_t dx = 0$$

recall that the constant  $G=1$ . Using integration by parts twice we obtain,

$$(3.2) \quad \int_0^L \frac{d}{dt} \frac{1}{2} (z_t)^2 dx + z_t z_{xxx} \Big|_0^L - z_{xt} z_{xx} \Big|_0^L + \int_0^L z_{xx} z_{xxt} dx = 0$$



We know from boundary conditions 1.5a, and 1.5b that

$z_{xx}(0,t)=z_{xx}(L,t)=z_{xxx}(0,t)=z_{xxx}(L,t)=0$  which causes the second and third terms of 3.2

to go to 0. This leaves us with,

$$(3.3) \quad \int_0^L \frac{d}{dt} \frac{1}{2} (z_t)^2 dx + \int_0^L \frac{d}{dt} \frac{1}{2} (z_{xx})^2 dx = 0$$

Combining these integrals we get,

$$(3.4) \quad \frac{d}{dt} \frac{1}{2} \int_0^L [(z_t)^2 + (z_{xx})^2] dx = 0$$

such that,

$$(3.5) \quad \int_0^L [(z_t)^2 + (z_{xx})^2] dx = H$$

where  $H$  is a constant determined by the initial state of the beam.

Now with equation 3.5 as our starting point we will use inequalities to prove that  $z(x,t)$  is bounded. We know that equation 3.5 is true thus,

$$(3.6) \quad |z_t| \leq J \quad \text{for every } (x,t)$$

and,

$$(3.7) \quad |z_{xx}| \leq J \quad \text{for every } (x,t)$$

where  $J$  is some arbitrary constant. Now the following must hold as well,

$$(3.8) \quad |z_x(x_0)| = \left| \int_{\frac{L}{2}}^{x_0} z_{xx}(x) dx \right| \leq \left| x_0 - \frac{L}{2} \right| J \leq \frac{L}{2} J$$

where  $x_0$  is some point on the elastic curve of our beam and  $\frac{L}{2}$  is of course the center of the beam.

Using 3.8 we then know,

$$(3.9) \quad |z(x_0)| = \left| \int_{\bar{x}}^{x_0} z_x(x) dx \right| \leq |x_0 - \bar{x}| \frac{L}{2} J \leq \frac{L^2}{2} J$$

Therefore,  $z(x,t)$  is bounded.

The above result is good news. It tells us that the vibrations caused by our “maneuvering rocket” type forces will not increase without limit until the structure fails.

## CHAPTER 4

### CONCLUSIONS

It is clear that mathematical modeling is an invaluable tool in the analysis of space structures. It allows us to examine various scenarios on earth before any space ship ever leaves the ground. The benefits to be gained from using mathematical modeling are two-fold. First, it is much cheaper than actual testing of hardware. Mathematical modeling can be used repeatedly to work out problems and identify solutions. This allows scientists to focus and test only the best solutions, thus saving money and resources. The other main benefit is that mathematical modeling can be used when it would be unsafe or unfeasible to test an actual system.

The purpose of this thesis was to develop a mathematical model that could be used to analyze the responses of a large space station type structure to the forces caused by the routine use of maneuvering rockets. Using differential equations a model, given by equation 1.8, was developed. A few solutions were approximated and coefficients found. Analysis then showed the vibrations caused by the maneuvering rockets were bounded. The results of this thesis can now be used in concert to examine specific various large space structure scenarios(i.e. different materials, different sizes, and masses). Below is a list of different materials used in the construction of space structures [7],

Table 4-1

<b>MATERIAL</b>	<b>DENSITY , <math>\rho</math> (Kg/m<sup>3</sup>)</b>	<b>Young's Modulus, E (N/mm<sup>2</sup>)</b>
Aluminum Alloy		
Sheet (2024-T36)	2770	72,000
Sheet (7075-T6)	2800	71,000
Beryllium Extrusion	1850	290,000
Magnesium Sheet	1770	45,000
Steel PH15-7 (RH1050)	7670	200,000

Data like this can be input into the model to analyze the response of specific materials to the forces caused by maneuvering rockets. Additionally, this model can be modified to examine different forces as well. For instance the case of the shuttle being used to boost a large space structure to higher orbit can be investigated by modifying this model.

In conclusion, as our government experiences shrinking research and development budgets it is clear that mathematical modeling will become increasingly important. The model developed in this thesis is only the beginning of what can be accomplished to ensure that mankind's colonization of space is safe and successful.

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